Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2021

Subject Name : Linear Algebra - I

Subject Code: 4SC03LIA1 **Branch: B.Sc. (Mathematics)**

Semester: 3 **Date: 15/12/2021** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:
1. Use of Programmable calculator & any other electronic instrument is prohibited.
2. Instructions written on main answer book are strictly to be obeyed.
3. Draw neat diagrams and figures (if necessary) at right places.
4. Assume suitable data if needed.

Q-1 **Attempt the following questions:** **[14]**

a) True or False : Intersection of two subspace is also subspace. (01)

b) Dimension of P_1 is _____ . (01)
(a) 0 (b) 1 (c) 2 (d) 3

c) For which value of $k$the vectors $(1,0,0)$, $(0,2,0)$, $(0,0,k)$ are linearly independent? (01)
(a) 0 (b) 1 (c) 2 (d) 1 & 2 both

d) If $V(F)$ is vector space then which of the following statement is false? (01)
(a) $V(F)$ is closed under vector addition
(b) $V(F)$ is closed under scalar multiplication
(c) Every element of $V(F)$ has multiplicative inverse
(d) every element of $V(F)$ has additive inverse

e) if $V(F) = R^2(R)$ then the additive inverse of $(3,2)$ is _____ . (01)
(a) (-3,2) (b) (-3,-2) (c) (1/3,1/2) (d) all

f) For bijective map $T: R^2 \to R^2$ then the rank of $T$is _____ . (01)
(a) 1 (b) 2 (c) 3 (d) 4

g) Find the value of k for which the vectors $(1,0,0)$, $(0, k-1, 0)$, $(0,0,k)$ are linearly dependent. (02)

h) Define subspace of vector space. (02)

i) Define Linear Transformation. (02)

j) Find the $d(u,v)$ if $u = (u_1, u_2) = (5,4)$& $v = (v_1, v_2) = (1,5)$. (02)

Attempt any four questions from Q-2 to Q-8 .

Q-2 **Attempt all questions** **[14]**

a) Show that $v = (-1,1,0)$ is liner combination of vectors $v_1 = (1,0,1)$, $v_2 = (-2,3,2)$, $v_3 = (-6,7,5)$. (05)

b) Prove:A non-empty subset W of vector space $V(F)$ is subspace of $V(F)$if and only if $\alpha u + \beta v \in W \ \forall \alpha, \beta \in F$ and $\forall u, v \in V$. (05)

c) Define $T: R^2 \to R^2$by$T(x,y) = (x + 2y, 3x - y)$.Show that the given transformation is linear. (04)

Q-3 **Attempt all questions** **[14]**

a) Prove that the set $S = \{(1,2,1), (2,1,1), 1,1,2)$ is basis for R^3 (06)

b) Express $v_1 = (3,4,6)$ as linear combination of $v_1 = (1,-2,2)$, $v_2 = (0,3,4)$, $v_3 = (1,2,-1)$. (04)

c) Let $T: R^2 \rightarrow R$ be linear map with $T(1,1) = 3$ and $T(0,1) = -2$. Then find $T(1,2)$. (04)

Q-4 **Attempt all questions** **[14]**

a) Let $T: V \rightarrow W$ be a linear transformation then show that Rank of T is subspace of W (05)

b) Let $T: R^2 \rightarrow R^2$ be linear transformation given by $T(x, y) = (x + y, x - y)$ then find T^{-1}. (05)

c) Find cosine angle between $u = (1,2)$ and $v = (0,1)$, also verify Cauchy-Schwarz Inequality. (04)

Q-5 **Attempt all questions** **[14]**

a) Let $V = R^+$, the set of all positive real numbers. Define the "sum" of two elements u and v in V to be their product i.e $u + v = u \cdot v$ and define "multiplication" of elements u in V by scalar α to be $\alpha u = u^\alpha$ (06)
Prove that V is real vector space with 1 as the additive identity.

b) Let $T: R^3 \rightarrow R^3$ be a linear transformation defined by $T(x, y, z) = (x + 2y + z, 2x - y, 2y + z)$. Find the matrix representation of T with respect to basis B, where $B = \{(1,0,0), (0,1,0), (0,0,1)$. (04)

c) Prove that $< u, v >= 3u_1v_1 + 2u_2v_2$ is an inner product space, where $u = (u_1, v_1)$ and $v = (v_1, v_2)$, $u, v \in R^2$ (04)

Q-6 **Attempt all questions** **[14]**

a) Which of the following set of vectors in vector space $V = R^3$ are linearly dependent or linearly independent? (05)

i) $S_1 = \{(4,-1,2),)(-4,10,2), (4,0,1) \}$

ii) $S_2 = \{(1,0,0), (0,1,0), (0,0,1) \}$

b) Show that the intersection of two subspace of vector space V is also subspace of V. (05)

c) Let $S = \{v_1, v_2\}$ be a subset of vector space $V(F)$ if S is linearly independent then show that $B = \{v_1 + v_2, v_1 - v_2\}$ is also linearly independent. (04)

Q-7 **Attempt all questions** **[14]**

a) Define a basis for vector space and show that $S = \{(1,0), (0,1)\}$ is basis for R^2. (05)

b) If $W = \{(x, y, z) \mid x - 3y + 4z = 0 \ \& \ x, y, z \in R\}$ prove that W is subspace of R^3. (05)

c) Explain : Reflection operator. (04)

Q-8 **Attempt all questions** **[14]**

a) Show that $V(F) = R^2$ is a vector space under operation defined as (07)
For $u, v \in R^2$ where $u = (u_1, v_1)$ and $v = (v_1, v_2)$ such that

$$u + v = (u_1 + v_1, u_2 + v_2) \ and \ \alpha(u_1, u_2) = (\alpha u_1 + \alpha u_2) \ \forall \ \alpha \in R$$

b) State and prove Rank-nullity theorem. (07)